MAC-CPTM Situations Project

Situation 37: Distributing Exponents

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Prompt

The following vignette took place in a high school Algebra 1 class. Most of the students were sophomores or juniors repeating the course. During the spring semester, the teacher had them do the following two problems for a warm-up:

1) Are these two expressions, $\left(x^3y^5\right)^2$ and x^6y^{10}, equivalent? Why or why not?
2) Are these two expressions, $\left(a+b\right)^2$ and a^2+b^2, equivalent? Why or why not?

Roughly a third of the class stated that both pairs of expressions were equivalent because of the distributive property.

Commentary

Mathematical Foci

Mathematical Focus 1

The students in this vignette appear to be over-generalizing the distributive property. They appear to be applying a procedure, *taking the number on the outside of the parentheses and multiplying it with what is inside of the parentheses*, and applying it where the property is not applicable.

Therefore, one focus for this vignette is proper application of the distributive property.

Mathematical Focus 2

The students in this vignette do not have a deep understanding of what an exponent is.

It may be the case here that the students have an understanding that m^3 means $m \cdot m \cdot m$-- but only when the base is a constant or a single variable. When the base is more complicated and parentheses are included in the expression, as illustrated by $\left(x^3y^5\right)^2$ and $(a+b)^2$, they may be enticed by the parentheses and the appearance of a number outside of the parentheses to apply what they recalled of a learned procedure. *Take the number on the outside of the parentheses and multiplying it with what is inside of the parentheses*.

Therefore, a second possible focus for this vignette is the meaning of exponents and expressions equivalent to $(A)^k$ where *A* is more complex than a simple constant and a single variable raised to a power.

For problem 1, $\left(x^3y^5\right)^2$ means "$\left(x^3y^5\right)$ times $\left(x^3y^5\right)$." Therefore, we apply the law of exponents for products ($m^a \cdot m^b = m^{a+b}$) to show $\left(x^3y^5\right)^2$ and x^6y^{10} are equivalent expressions.

$$\left(x^3y^5\right)^2 = \left(x^3y^5\right)\left(x^3y^5\right) = x^3x^3y^5y^5 = x^6y^{10}.$$

For problem 2, $(a+b)^2$ means "$(a+b)$ times $(a+b)$." Since we have the product of two binomials, a different law is needed to carry out the multiplication.

$$(a+b)^2 = (a+b)(a+b) = a(a+b)+b(a+b) = a^2+ab+ba+b^2 = a^2+2ab+b^2$$

Although students may have thought they were applying the distributive property to show that $(a+b)^2$ and a^2+b^2 were equivalent, it is because of the distributive property of multiplication with respect to addition that $(a+b)^2$ and a^2+b^2 <u>are not</u> equivalent.

References

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